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Bayesian Approach to the Decision of tasks of Insurance and Warranty Service

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Abstract - Insurance activities play an important role in the development of the country's economy. Practice shows that in the market of insurance services there are deviations of actual payments from their planned values determined in the sizes of tariff rates. As a result, insurance companies are forced to hold a significant amount of funds in liquid assets to compensate for unaccounted volatility, which reduces the effectiveness of insurance activities or increase the size of tariff rates, which affects the availability of insurance services. The possibility to take into account such deviations will allow insurance companies to approach the calculation of insurance tariffs more carefully and to conduct their activities more efficiently, which ultimately will positively affect the state of the insurance industry and the economy of the country as a whole. From this point of view, minimizing the risk of ruining an insurance company, evaluating and optimizing the parameters of warranty obligations are important tasks in decision theory.This paper investigates the problem of insurance and warranty services in a certain class of the probability distribution as a priori in this situation.

It applies Bayesian approach - the direction of the management, which is based on the principle of maximum use of the available a priori information about the processes occurring in the enterprise and in the external environment, its continuous, taking into account received sample data review and re-evaluation.

Keywords - a priori distribution, Beta – binomial distribution, Bayesian Approach, Bayesian Belief Network, probability of ruin, Gamma distribution, model of individual insurance

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1 Introduction

Many large-scale control systems operating in conditions of great uncertainty and fuzziness of information. In conditions of uncertainty the possibility of undesirable events associated with the infliction of economic, moral or other type of damage. These events are characterized by the risk of their implementation. For a significant number of areas of expertise is a general understanding the representation of risk as a characteristic of the adverse effects of the uncertainty of the situation. In such areas for reducing impacts of risk is using methods of risk management, which include the definition and measurement of risks, risk assessment and management.

Risk management as a decision-making process aimed at reducing the adverse consequences of risky situations, aimed at increasing economic efficiency of the institution.

As part of this article the Bayesian approach to risk assessment of the insurance companies and the parameters of the warranty examines.

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The idea Bayesian approach[1] consists of a transition from a priori knowledge of the system to a posteriori knowledge in view of taking into account the observed parameters involved in the calculations. It is used in the solution of problems of classification [2], inventory control theory [3], insurance and warranty service.

The key concept in the application of Bayesian approach is Bayesian network or Bayesian Belief Network (BBN) - a probabilistic model, which is a set of variables and their probabilistic dependencies. BBN device allows to combine the available statistics on the characteristics of the process under study, in addition to expert information provided by experts. Most of the diagnostic models to support decision-making on the basis of BBN is based on expert information [4], but the expert information is often insufficient to build a complete model. An important advantage of the model, based on the principles of artificial intelligence, machine learning is the ability to model structure, that is, even if the original structure of the model was incomplete, it is possible to improve the model by using the incoming data. Moreover, due to insufficient data model, in some cases, apply the approximate evaluation probabilistic inference in the diagnosis Bayesian network.

The structure of the Bayesian Belief Network represents an element considered random process and the relationships between them. The Bayesian approach all values and parameters are considered random because even if they have specific values, exact distribution laws are unknown. Rate of unknown parameter of the system means to find the posterior distribution.

2 Theoretical provisions for investigation tasks

For effective management of institutions, we must first have a complete, accurate and timely information on the state of the system. Main gaps in knowledge related to the complexity of the various systems. Suppose that we observe some random variable that has a probability density with parameter θ. Before the advent of any statistical data or observations about the parameters are often not known. Therefore if we ignore the requirement of simplicity of calculations, the preferred, or "good" prior distribution should minimally affect the conclusion that on the posterior distribution, as well as to take into account the absence of a priori information about the parameters. A priori distribution modeling the lack of information, called uninformative.

The postulate Bayes-Laplace says that when it is not known in advance about the parameter θ, a priori distribution should be taken uniform, ie all possible outcomes of a random variable θ have equal probability. The main problem of the use of uniform distribution as a non-informative prior distribution is that a uniform distribution is not invariant with respect to the function of the parameter. If we know nothing about the parameter θ, we also do not know, for example, the function of 1 / θ. However, if θ is uniformly distributed, then 1 / θ longer it has a uniform distribution, although according to postulate Bayes-Laplace 1 / θ should have a uniform distribution. Moreover, uniform distribution can not be used as a priori, if the set of parameter values is infinite. We should also note the significant dependence of uniform distribution on a variety of outcomes. One can cite the following example to illustrate this independence. Suppose that in a poke there are balls of red color and some other colors. It is unknown how many balls as the red and all the other balls in a poke. What is the probability that the first ball of pulling out of the poke is red? Assuming uniform distribution, we obtain the probability of 1/2. Now suppose that in a poke, there are the balls of red, green, and some others. The probability of a red ball is 1/3. The situation in the second case has not changed since the green balls can be attributed to the balls' of a different color "as applied to the first situation. However, we have very different probabilities of the red ball.

In the literature, there are quite a number of approaches for the selection of a non-informative prior distribution having its advantages and disadvantages. However, the most interesting is the approach that is completely different from most traditional. The essence of this approach is as follows. Define more than one prior distribution, and the whole class $\mathcal{M}$ distributions π, for which you can find lower and upper probability of the event A as

$$\underline{P}(A) = inf\{P_\pi(A): \pi \in \mathcal{M}\} \quad (1)$$
$$\overline{P}(A) = sup\{P_\pi(A): \pi \in \mathcal{M}\}$$

Under certain conditions, the set $\mathcal{M}$is completely determined by the lower and upper probability distribution functions. It should be noted that the class $\mathcal{M}$ to be regarded "as a class is not suitable prior distributions, as well as a suitable class of prior distributions." This means that the individual distribution of the class is not the best or "good" a priori distribution, since no single distribution can not satisfactorily simulate the absence of information. But the whole class as a whole, define the upper and the lower the probability distribution is a suitable model to lack of information. This means that the individual distribution of the class is not the best or "good" a priori distribution, since no single distribution can not satisfactorily simulate the absence of information. But the whole class as a whole, define the upper and the lower the probability distribution is a suitable model to lack of information. But the whole class as a whole, define the upper and the lower the probability distribution is a suitable model to lack of information. When almost no a priori information, $\underline{P}(A)$for this class should be close to 0, and $\overline{P}(A)$is close to 1. This means that the event may be a priori any probability. In most cases, the class of distributions $\mathcal{M}$parametrically defined. Therefore, minimum and maximum (1) are usually calculated according to some parameters.

A large number of applications related to the necessity of determining the probability that for a certain observation period not occur or at least not more than a predetermined amount of defined events. If you know the probability θ of the event, the probability $p(k|\theta)$ what happens $k$event possible, is based on the binomial distribution

$$p(k|\theta) = C_n^k \theta^k (1-\theta)^{n-k} \quad (2)$$

If the probability is not known exactly, and she is a random number with a certain probability density function $\pi(\theta|\upsilon)$, then the probability of what happened exactly k events is determined by the expression

$$P(k) = \int_\Omega p(k|\theta)\,\pi(\theta|\upsilon)d\theta$$

Here υ - vector of distributions parameter π; $\Omega = [0,\ 1]$ set of parameter values θ.

In this case, to agree a priori and a posteriori distributions as a function of π using beta distribution. The prior beta distribution of the random variable θ, denoted $Beta\ (a, b)$, has a probability density function.

$$\pi(\theta|a, b) = \text{Beta}\ (a, b) = \frac{1}{B(a, b)}\theta^{\alpha-1}(1-\theta)^{b-1},\ 0 \le \theta \le 1.$$

Where $a > 0, b > 0$ - distribution parameters, ie, $\upsilon = (a, b)$; $B(a, b)$ - beta function.

It should be noted that beta-distribution is used as a priori in this situation the search for the likelihood that the $k$event will occur. In contrast, the previously considered a special case of the Dirichlet distribution is used to find the probability that there will be exactly one event.
Suppose that as a result of the observations from the total number of N observations occurred interested us Kevents. Then the posterior beta - distribution $\pi(\theta|\upsilon, K)$subject to the statistical data is as follows:

$$\pi(\theta|\upsilon, K) = Beta\ (a + K, b + N - K).$$

From here, the probability $P(k)$what would happen exactly $k$defined events in the future with total number of n observations is:

$$P(k) = \int_\Omega p(k|\theta) \cdot \pi(\theta|\upsilon, K) d\theta =$$

$$= \int_0^1 C_n^k\, \theta^k (1-\theta)^{n-k} \cdot$$

$$\cdot Beta\ (a + K, b + N - K) d\theta =$$

$$= C_n^k \frac{B(a + k + K, b + n + N - k - K)}{B(a + K, b + N - K)}$$

This is beta binomial distribution. The probability that the number of interested events will not exceed$M$, is

$$P_M = \sum_{k=0}^M C_n^k \frac{B(a+k+K, b+n+N-k-K)}{B(a+K, b+N-K)} \qquad (3)$$

If we consider the number of defined events for a certain period of time t, one of the most common distribution laws to describe the corresponding probabilities is the Poisson distribution with parameter λcharacterizing the intensity of the events. The probability of the $k$event in a time t in accordance with the Poisson distribution is as follows:

$$p(k) = \frac{(\lambda t)^k \exp(-\lambda t)}{k!}$$

If the intensity of the events λ is not known exactly, then assuming that it is a random number with a certain probability density function π (λ|θ), you can use a Bayesian approach. At the same time to agree a priori and a posteriori distribution function π must comply with the density of the gamma distribution. Priori gamma-distributed random variables λ, denoted by a probability density Gamma (a, b)

$$\pi(\lambda) = Gamma\ (a, b) =$$
$$= \frac{1}{\Gamma(a)} b^a \lambda^{a-1} \exp(-b\lambda), \lambda > 0$$

Here $a > 0, b > 0$- the parameters of the distribution, i.e, $\vartheta = (a, b)$; $\Gamma(a)$ - Gamma - function.

Let during the $n$observation period in the past occurred in the amount $K = \sum_i^n k_i$of events. Here k_i- the number of events on the i - th observation period. Then the posterior probability density function $\pi(\lambda|K)$of the gamma distribution, subject to the statistical data in the form of Kevents during the $n$observation period is as follows:

$$\pi(\lambda|K) = Gamma\ (a + K, b + N)$$

From here probability $P(k)$that will happen exactly in the future provided $t = 1$ is:

From here probability that exactly the $k$events will occur in the future provided is of the form:

$$P(k) = \int_0^\propto p(k|\lambda) \cdot \pi(\lambda|\vartheta)\, d\lambda =$$

$$= \int_0^\propto \frac{\lambda^k \exp(-\lambda)}{k!} \cdot Gamma(a, b) \cdot d\lambda =$$

$$= \frac{\Gamma(a + k)}{\Gamma(a) k!} \cdot \left(\frac{b}{b+1}\right)^a \left(\frac{1}{b+1}\right)^k$$

This is the negative binomial distribution. From here probability that the number of interested events exceeds M is

$$P_M = \sum_{k=0}^M \frac{\Gamma(a + k)}{\Gamma(a) k!} \cdot \left(\frac{b}{b+1}\right)^a \left(\frac{1}{b+1}\right)^k$$

The probability $P(k)$is also determined using the recurrence formula

$$P(k) = \begin{cases} \left(\frac{b}{b+1}\right)^a, & k = 0 \\ \frac{a + k - 1}{k(b + 1)} \cdot P(k-1), & k \geq 1 \end{cases}$$

Consider the case when the previous $n$observation periods are not the same and equal to$t_1, \ldots, t_n$. At the same time the total observation time is $T = t_1 + \ldots + t_n$. The number of events observed in each period is equal to $k_1, \ldots, k_n$ and their total number of T times equal to $k_1 + \ldots + k_n$. It is more common and interesting case, since in practice it is difficult to expect that the initial statistical data collected during the same time periods as the forecast period t.

The probability the k event in t time, provided that during the observation t_1 period occurred k_1 events is

$$P(k) = \int_0^\infty \frac{(\lambda t)^k \exp \lambda t}{k!} \cdot Gamma\ (a + k_1, b + t_1) d\lambda =$$

$$= \frac{\Gamma(\mathrm{a} + k_1 + \mathrm{k})}{\Gamma(a + k_1)k!} \cdot \left(\frac{b + t_1}{b + t_1 + t}\right)^{a+k_1} \left(\frac{t}{b + t_1 + t}\right)^k$$

A similar expression for $P(k)$ can be obtained for an arbitrary value $n \geq 1$. At the same time, as the resulting expression obtained probability does not depend on the distribution of the number of events $k_1, \ldots, k_n$ and $t_1, \ldots, t_n$ on the distribution of the observation time, and is determined only by the total number of K events and total time T of observation, i.e

$$P(k) = \frac{\Gamma(\mathrm{a} + \mathrm{K} + \mathrm{k})}{\Gamma(a + K)k!} \cdot \left(\frac{b + T}{b + T + t}\right)^{a+K} \left(\frac{t}{b + T + t}\right)^k$$

In the special case when all time slots are identical and have unit length, the probability of k events per unit time is of the form

$$P(k) = \frac{\Gamma(\mathrm{a} + \mathrm{K} + \mathrm{k})}{\Gamma(a + K)k!} \cdot \left(\frac{b + n}{b + n + 1}\right)^{a+K} \left(\frac{1}{b + n + 1}\right)^k$$

The probability P_M is defined as

$$P_M = \sum_{k=0}^{M} \frac{\Gamma(\mathrm{a+K+k})}{\Gamma(a+K)k!} \cdot \left(\frac{b+T}{b+T+t}\right)^{a+K} \left(\frac{t}{b+T+t}\right)^k \quad (4)$$

The negative binomial distribution with parameters $a = 1, b = 1$ is a non-informative prior distributions.

3 The Probabilistic Model of the Insurance company

The probabilistic behavior of the insurance market and of the insurance companies described in many economic and mathematical models [5]. A common requirement for all such models is theavailability of information on premiums, the insurance cases and payment of damages. This information is sufficient to construct a model of the rough total X of the insurance company and the conclusion of management contracts:

$$X = C - R,$$

where C- collected premiums, the value of a deterministic, R - the cumulative payments. The stochastic nature of R does not allow one to predict the result of the X, and to study the various possible outcomes natural to use the concept of "average results", "deviation from the mean", "the possibility of exceeding the value of $R$the value C". A more meaningful and adequate models require understanding of the structure C, R, external economic environment in general and the financial market in particular.

From all variety of models further considered only the basic and least complicated mathematically, which together with the natural economic meaning allows demonstration of the probabilistic methods.

The base model of insurance are the following objects:

- x - initial capital
- t - time parameter, usual $t \in \mathbb{R}^+$ or $t \in \mathbb{Z}^+$
- $0 \leq \sigma_1 \leq \cdots \sigma_n \leq \cdots$ - successive moments of insurance payments
- $N(t) = \sup\{n\colon \sigma_n \leq t\}$ - the total number of payments by the time t, $N(t) = 0$
- If { $\sigma_1 > t$, for $n \geq 1$ it is rightly the relation $\{N(t) = n\} = \{\sigma_n \leq t \leq \sigma_{n+1}\}$
- a sequence of independent and identically distributed random variables $\{y_k\}$, $y_k -$ insurance payment at the time σ_k (k-th jump $N(t)$)
- $R(t) = \sum_{k=1}^{N(t)} y_k -$ process risk denotes the the total payments to the moment t
- $\Pi(t)$ - collected at the time t of premiums
- $X(t) = x + \Pi(t) - R(t) -$ capital

If $t \in \mathbb{R}^+$ we speak of continuous time models. If $t \in \mathbb{Z}^+$ we speak of a model of discrete time and the time parameter to represent and clarity. We believe that all these processes are set at some fixed space $(\Omega, \mathfrak{F}, \mathbb{P})$.

If a particular implementation of the process $X(t)$it may happen that at some moment of time sum $\tau$to payment exceeds the current capital $X(\tau^{-}) = \lim_{s\uparrow\tau} X(s)$, while formally have the time $X(\tau) < 0$ and called τ the moment of ruin.

Such moments may be few and assume that τ- moment of the first ruin. In such cases, the company must to seek additional resources to pay, the frequent repetition of similar situations - a signal to the revision of the formation of premiums $\Pi(t)$.

The values $\psi(x) = \mathbb{P}\{\tau <\propto\}$, $\psi(x,\ t) = \mathbb{P}\{\tau \le t\}$, called the probability of ruin on a finite and infinite time interval, $\varphi(x) = 1 - \psi(x), \varphi(x,t) = 1 - \psi(x,t)$ - the probability of non-bankruptcy.

Depending on the initial capital and time indicated in the methodological purposes. Of course, the probability of non-bankruptcy also depends on the structure $\Pi(t), R(t)$, but because of the tradition studies dependence and it is from x and t. You can understand the scheme itself as: entry into the insurance business with the parameters $\{\Pi(t), R(t)\}$ ensures reliability $\varphi(x,t)$at a finite $[0,\ t]$and reliability $\varphi(x)$ on an infinite time interval. Note the obvious properties:

ϕ(x)=P{X(t)≥0 for all t}

$$\varphi(x,t) = \mathbb{P}\{X(s) \ge 0 \text{ for all } \ s \le t\}$$

$\varphi(x), \varphi(x,t)$increases by x, ($\psi(x), \psi(x,t)$decreases by x), "more capital - more reliability", $\varphi(x,t)$decreases by t to$\varphi(x)$($\psi(x,t)$increases by t to $\psi(x)$), "more risky period - less reliability"

$$\varphi(x,t) \le \mathbb{P}\{X(t) \ge 0\} = \mathbb{P}\{R(t) \le x + \Pi(t)\} \quad (5)$$

The distribution function of the risk process

$$\mathbb{P}\{R(t) \le t\} = \sum_{k=1}^{\propto} \mathbb{P}\{N(t) = k\} \cdot \cdot \mathbb{P}\{y_1 + \cdots + y_k \le x\} \quad (6)$$

Simplifying the right-hand side of this equality is possible only if a more specific description of the process $N(t)$and clarification distribution $\{y_k\}$.

Formation of the of premiums happening on the the basis of knowledge about the process of the upcoming payments $R(t)$ and follows formally to write $\Pi\big(t, R(t)\big)$,entry $R(t)$ is used for compactness. Mandatory condition activity of the insurance company's the work of "not at a loss»

$$E\Pi(t) > ER(t) \quad (7)$$

This means, that during the time $t$of work the company in average will receive regular income $E\Pi(t) - ER(t)$.

*Model individual risk.*The random nature insurance cases does not allow one to determine the parameters of insurance and there is always some risk of bankruptcy of the insurance company. Actuarial calculations are designed to determine the parameters of of insurance so as to minimize the risk of bankruptcy, or limit it to some limiting value.

The model of individual risk is determined by by the following relations:

$$\Pi(n) = c \cdot N$$

$$R(n) = \begin{cases} 0 & , \quad n < n_0 \\ X_1 + \cdots + X_N, & n = n_0 \end{cases} \quad (8)$$

The random variables X_i are independent.

This situation is typical for the case when the company enters into Nsimilar contracts with equal period of validity and the possibility of paying for i- mu contract premium X_i, for each contract is equal c.In this comparatively small amount of time, and the interest is only the final financial result. Indeed, in this case, the probability of non-bankruptcy

$$\varphi(x,n) = \begin{cases} 1 & n < n_0 \\ \mathbb{P}\{c \cdot N - X_1 - \cdots - X_N \ge 0\} & n = n_0 \end{cases} \quad (9)$$

Arising theory here is adapted for solving "local" problems. The mathematical apparatus is the sum of independent random variables.

Consider the case of $N = 1$, capital $X = c - X_1$. The value $X_1$is the result of the influence of two types of chance, which are naturally assumed to be independent: the fact of the occurrence or non-occurrence of insured event and the actual value of the payment.

Therefore, we can write $X_1 = I_1 \cdot y_1, I_1$- an indicatorthe occurrence of the insured event, $y_1 > 0$ - the size of the payment. Let $\mathbb{P}\{I_1 = 1\} = q$, In this case, the probability of non-bankruptcy is defined in the following methods:

I method.

$$P_M = P\{\Pi(t) \ge R(t)\} = P\{cn \ge X_1 + \cdots + X_n\} \quad (10)$$

If the random variables $X_1, \dots, X_n$are independent, and the values of parameters $c$and$n$ are fixed, $y_i = y$for all

$i=1,\dots,n$, the problem of determining the probability of non-bankruptcy simply solved by the introduction of the random variable $K$of number insurance cases, which has a binomial distribution $p(k|\theta)$ from (2) with parameter θ. Thus the probability of non-bankruptcy is

$$P_M=\sum_{k=0}^{M}p(k|\theta)$$

Here $M$is the maximum number of actions that the company can pay, defined as

$$M=\lfloor \Pi(t)/y\rfloor=\lfloor cn/y\rfloor, \quad (11)$$

Symbol $\lfloor\cdot\rfloor$means the integer part "from below".
As seen from the description of the model, the probability of non-bankruptcy can be calculated using the Bayesian approach, namely the use of beta binomial distribution.
II method.$EX_1=q\cdot Ey_1$

$$DX_1=q\cdot Ey_1^2-q^2\cdot(Ey_1)^2,$$

$$Ee^{r\cdot X_1}=1-q+q\cdot Ee^{r\cdot y_1}.$$

Next, we describe several methods of calculating premiums for arbitrary N.

If you know the function of distribution of risk $\mathrm{F}_N(x)=\mathbb{P}\{X_1+\dots+X_N\le x\}$, then

$$\varphi(x,n_0)=\mathrm{F}_N(c\cdot N)$$

continue to apply the standard scheme: fixed some level of risk ε (the small number) and assumed

$$c=\frac{\mathrm{F}_N^{-1}(1-\varepsilon)}{N}$$

In this case $\varphi(x,n_0)=1-\varepsilon$, it means that with the probability of premiums $1-\varepsilon$will be enough for the final payment actions.

If the function $\mathrm{F}_N(x)$is complex to access the form, it is used to calculate $\varphi(x,n_0)$the approximate methods. The first method is the central limit theorem, according to which a fraction

$$\frac{X_1+\dots+X_N-N\cdot(q\cdot Ey_1)}{\sqrt{N\cdot(q\cdot Ey_1^2-q^2\cdot(Ey_1)^2)}}$$

distributed "approximately" on the standard normal distribution $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-x^2/2}\,dx$. Inthiscase

$$\varphi(x,n_0)\approx\Phi\left(\frac{c\cdot N-N\cdot(q\cdot Ey_1)}{\sqrt{N\cdot(q\cdot Ey_1^2-q^2\cdot(Ey_1)^2)}}\right)$$

And the level of risk can be put

$$c=\ q\cdot Ey_1+\Phi^{-1}(1-\varepsilon)\cdot \\ \cdot\sqrt{\left(\frac{q\cdot Ey_1^2-q^2\cdot(Ey_1)^2}{N}\right)}$$

It is also used estimate "from below" of the probability of non-bankruptcy. Ifforsome r>0 issatisfied

$Ee^{r\cdot y_1}=1$, then$Ee^{r\cdot X_1}=1, Ee^{r\cdot(X_1+\dots+X_N)}=1$ and

$$\varphi(x,n_0)=\mathbb{P}\{X_1+\dots+X_N\le c\cdot N\}= \\ =\mathbb{P}\left\{e^{r(X_1+\dots+X_N)}\le e^{r\cdot(c\cdot N)}\right\}\ge \\ \ge 1-e^{-r\cdot(c\cdot N)}$$

In this case, the level of risk ε can be put

$$c=\frac{\ln(1/\varepsilon)}{r\cdot N}$$

4 Probabilistic Model of Warranty

Consider the use of the negative binomial distribution model warranty. Production, procurement and sale of goods is generally linked to the risk that part of the product after purchase or during the operation fails, refuses or unable to perform some of its functions. One way to minimize or compensate for this risk are the warranty.
In accordance with the manufacturer's warranty specifies the time during which the faulty product will be repaired, replaced or the buyer reimbursed damage. While under warranty, the amount of damages, the cost of the goods, their quantity, etc. may be considered as parameters warranty. The values of the parameters are chosen so that the losses due to failure of the product in a sense do not exceed incomes from the production and sale of these products. Therefore wrong estimated parameters warranty may result in significant losses for the producers and the seller.
Evaluation and optimization of parameters of the warranty is one of the important problems of the theory of decision-making. As in any decision-making problem, the optimal solution, or the risk mistaken decision fully determined by of the available information on the state of nature, which in turn depends on knowledge of the probability distribution defined on the states of nature. In terms of the warranty it is

necessary to know the probability characteristics of the reliability of the products or parameters of probability distributions, such as the number of failed product in the predetermined time.
Consider the flow of goods between producers and consumers. The following scheme warranty is sufficiently typical. Let n- the number of product that the buyer would like to purchase.You can also consider final buyer, but then $n = 1$, and it is a special case of the general scheme. Assume that all of these products are the same. On each product installed warranty period t (time units). The buyer prepared to pay $x$for one product and not ready to claim damages failure no more $z$product in the time interval $[0, t]$. However, failure of the product for each customer demands compensation over $z$monetary units per thefailed product. In the particular case, the parameter $z$is 0. The problem of optimizing the parameters of warranty from the point of view of the manufacturer is the optimum choice in a certain sense values $z$and y. Suppose that the production cost of one product is $c$units. Assume that the cost of production of one product is k units. Then in case of failure $z$or less products for the time $t$manufacturer's profit is $B = n\,(x - c)$. However, if $i > z$ failures occurred during t manufacturing, the manufacturer is obliged to pay for warranties $(i - z)y$ and profit is

$$B = n(x - c) - (i - z)y$$

We remark that most important parameter is the warranty - period t. Therefore, to describe the flow of failures during this time, and to calculate the probability $p(i|\lambda)$ of $i$failures for this time Poisson distribution with parameter λ is most preferred. Necessary to note that the Binomial distribution may also be used herein to describe the probability of failures.

Expected payout in accordance with the warranty, can be considered as mathematical expectation

$$\sum_{i=z+1}^{n} y(i - z)p(i|\lambda)$$

From here the expected income producer for a given distribution $p(i|\lambda)$ is determined as the difference between the profit $n(x - c)$from the sale of n products in view of their prices, the cost and the expected payments under the guarantee, ie

$$\mathbb{E}_p B = n(x - c) - \sum_{i=z+1}^{n} y(i - z)\, p(i|\lambda)$$

Thus, the expected income depends on the intensity of failures λ. If the expected income is greater than 0, the warranty can be considered as unprofitable and effective. Otherwise, they should be reconsidered in favor producer.

Note the scheme is close to the warranty insurance scheme, but in contrast to the objective of the problem of insurance aim of this task is to calculate of mathematical expectation $\mathbb{E}_p B$, not the probability distribution function.
However, the scheme of evidence remains similar. Using the negative binomial distribution, it is easy to obtain the following expression for the expected income under certain parameters$(a,\ b)$, and provided that during the observation T of Kfailures occurred. From (4) we get:

$$\mathbb{E}_p B = n(x - c) - y \sum_{k=z+1}^{n} \frac{(\mathrm{k} - \mathrm{z})\Gamma(a + K + k)}{\Gamma(a + K)k!} \times$$
$$\times \left(\frac{b + T}{b + T + t}\right)^{a+K} \left(\frac{t}{b + T + t}\right)^{k}$$

5 The results of the experiments

Example 3.1 Let the insurance company has concluded a) $\mathrm{T} = 100$ contracts; b) $\mathrm{T} = 1000$ contracts. The probability of an insured event under one contract $q = 0{,}1$,in this case the paid amount $y = 100$. The premium of a single contract is $c = 12$. Find the probability of non-bankruptcy according to the method described.

The decision on the method I a): The maximum number of lawsuits that the company can pay equal

$$M = \lfloor cn/y \rfloor = 12 \cdot \frac{100}{100} = 12$$

Suppose that as a result of observation of the total number of $N = 20$ contracts before had $K = 1$ an accident. Then the probability of non-bankruptcy in the selection of parameters $a = 1$ and $b = 1$ non-informative prior distribution from (3) is

$$P_{12} = \sum_{k=0}^{12} \mathrm{C}_{100}^{k} \frac{B(a + k + 1, b + 100 + 20 - k - 1)}{B(a + 1, b + 20 - 1)} =$$

$$= \sum_{k=0}^{12} C_{100}^{k} \frac{B(1+k+1, 1+100+20-k-1)}{B(1+1, 1+20-1)} =$$
$$= 0.744$$

As can be seen from the results, evaluation of the probability of non-bankruptcy 0.744 too pessimistic with such a small sample of observed events in the past.

Suppose now that from 200 contracts in the past there have been 10 cases of insurance. Then it is $P_{12} = 0.986$. This indicates significant dependence on the calculated probabilities from a priori distribution with a small amount of statistical data.

The decision on the method II b): According to model of individual risk capital at the end of the term of the contract is written in the form

$X = 12 \cdot 1000 - X_1 - \cdots - X_{1000}$. The distribution X_i is of the form $X_i = 100 \cdot I_i$, $\mathbb{P}\{I_1 = 1\} = 0{,}1$, so

$$EX_1 = 0{,}1 \cdot 100 = 10,$$

$$DX_1 = 0{,}1 \cdot 10000 - 0{,}01 \cdot 10000 = 900.$$

Collected premiums constitute 12000, so the company can pay a maximum of 120 lawsuits and

$$\mathbb{P}\{X_1 + \cdots + X_N \leq c \cdot N\} =$$
$$= \sum_{k=0}^{120} C_{1000}^{k} \cdot (0{,}1)^k \cdot (0{,}9)^{1000-k} \approx 0{,}983$$

In fact, the probability of non-bankruptcy is equal to the probability no more than $\frac{12 \cdot 1000}{100} = 120$ lawsuits).Value is accurate to $0{,}001$.The use of the central limit theorem gives

$$\mathbb{P}\{X_1 + \cdots + X_N \leq c \cdot N\} \approx \Phi\left(\frac{12000 - 10000}{\sqrt{1000 \cdot 900}}\right)$$
$$= \Phi\left(\frac{20}{\sqrt{90}}\right) \approx 0{,}982$$

Example 3.2 Suppose that an insurance company has concluded $N = 1000$ contracts. The probability of an insured event under one contract $q = 0{,}1$, in this case is paid amount $y = 100$. Find premium of one contract, that the probability of ruin is not exceeded $0{,}95$.

Non-bankruptcy in this case means that the probability does not exceed $0{,}95$ the number of lawsuits $\frac{c \cdot 1000}{100} = 10 \cdot c$, where we have the equation

$$\sum_{k=0}^{10 \cdot c} C_{1000}^{k} \cdot (0{,}1)^k \cdot (0{,}9)^{1000-k} = 0{,}95,$$

difficult to find an exact solution. From the central limit theorem

$$\mathbb{P}\{c \cdot N \geq X_1 + \cdots + X_N\} \approx \Phi\left(\frac{1000 \cdot c - 10000}{\sqrt{1000 \cdot 900}}\right)$$

And the equation for c takes the form

$$\Phi\left(\frac{10 \cdot c - 100}{\sqrt{90}}\right) = 0{,}95$$

$$c \approx 10 + \sqrt{90} \cdot 0{,}164 \approx 11{,}6$$

For value $c = 11{,}6$ is accurate to $0{,}001$

$$\sum_{k=0}^{116} C_{1000}^{k} \cdot (0{,}1)^k \cdot (0{,}9)^{1000-k} \approx 0{,}957 > 0{,}95.$$

Note that for value $c = 11{,}5$ up to $0{,}001$ the probability of non-bankruptcy is approximately equal to $0{,}947$.

Example 3.3 How many contracts N must conclude an insurance company, to the probability of non-bankruptcy is not exceeded $0{,}95$. The probability of an insured event under one contract is $q = 0{,}1$, in this case amount is paid $y = 100$, premium for one contract - $c = 11$.

Substitutingintotheequationties

$$c = q \cdot Ey_1 + \Phi^{-1}(1-\varepsilon) \cdot$$
$$\cdot \sqrt{\left(\frac{q \cdot Ey_1^2 - q^2 \cdot (Ey_1)^2}{N}\right)}$$

numerical values, then $11 = 10 + 1{,}64 \cdot \frac{30}{\sqrt{N}}$,

$$N \approx 2421.$$

For this value will be a real probability of non-bankruptcy $0{,}949$.

We remark that estimate of the probability of non-bankruptcy from the central limit theorem increases monotonically in c, N.The real probability of non-bankruptcy increases in c, but the dependence on N, in general more complicated.

Example 4.1 Producer sells party products in the amount $n = 100$ of shares at a price per unit $x = 200\$$ at its cost $c = 160\$.$ Term of warranty - $t = 1$ year. In testing $K = 2$ products are out of order for a total observation period $T = 3.$ In case of refusal over $z = 1$ products damages is carried out. Find the optimal value for the payment of compensation of damages for each refused product.

We are using first an exact a priori negative binomial distribution with parameters $a = 1$ and $b = 1$. Then, theexpectedincomeis

$$\mathbb{E}_p B = 100 \cdot 40 - y \sum_{k=2}^{100} \frac{(\mathrm{k}-1)\Gamma(3+k)}{\Gamma(3)k!} \times$$
$$\times \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^k = 4000 - y \cdot 0.262$$

From here the condition of a positive the expected income is:

$$4000 - y \cdot 0.262 \geq 0$$

and$y \leq 15267$. This value indicates that the producer is willing to pay $15267\$$ for each product in the failed commodity unit during the year to have a positive the expected income.

6Conclusion

Insurance has always been associated with risks and their evaluation. Risk assessment, analysis of insurance markets, problems of various types of insurance, the relationship between insurance and public policy, the problem of training personnel for the insurance industry - all this is very important today. Discussion about the role of insurance in an innovative economy requires further development, primarily by discussing the results of research conducted in this direction, their interpretation and the development on their basis of new macroeconomic policy tools that are effective in an innovative economy [6].
At the present stage, the effective functioning of the insurance company can influence the innovative development of the economy from various aspects. Unfortunately, not many insurance companies are currently ready to insure innovative risks, since the domestic market is not sufficiently developed and there is weak competition in it, insurers are more profitable to work in already mastered market segments. In addition, there is no well-defined algorithm for insurance of innovative risks. The development of such an algorithm must be thoroughly studied and scientifically justified.

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